Classical Loop Nest Transformation Framework on MLIR

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Existing Loop Transforms in MLIR

- Works on Affine dialect operations
- No generic Analysis framework yet
  - Dependence analysis are local to loop nests
- No unified driver for all loop transforms
- Most transformations works only if all the loops in a loop nest are `AffineForOp`

Not all Affine loops can be converted to Affine Ops

- Custom Types may not be converted / “cast”ed to std.memrefs
  - Example: Array of structures

    ```
    %3 = fc.allocate  : !fc.ref<fc.array<10 x fc.struct_type<i32, f32>>>
    ```

  - **Different loop nest transformations for different types?**

- Memory Dependence analysis not just for Affine Ops
  - What happens to custom dialect operations inside Affine loops?
  - affine.store vs. std.store vs. vector.load vs. **fc.load (or any custom dialect)**

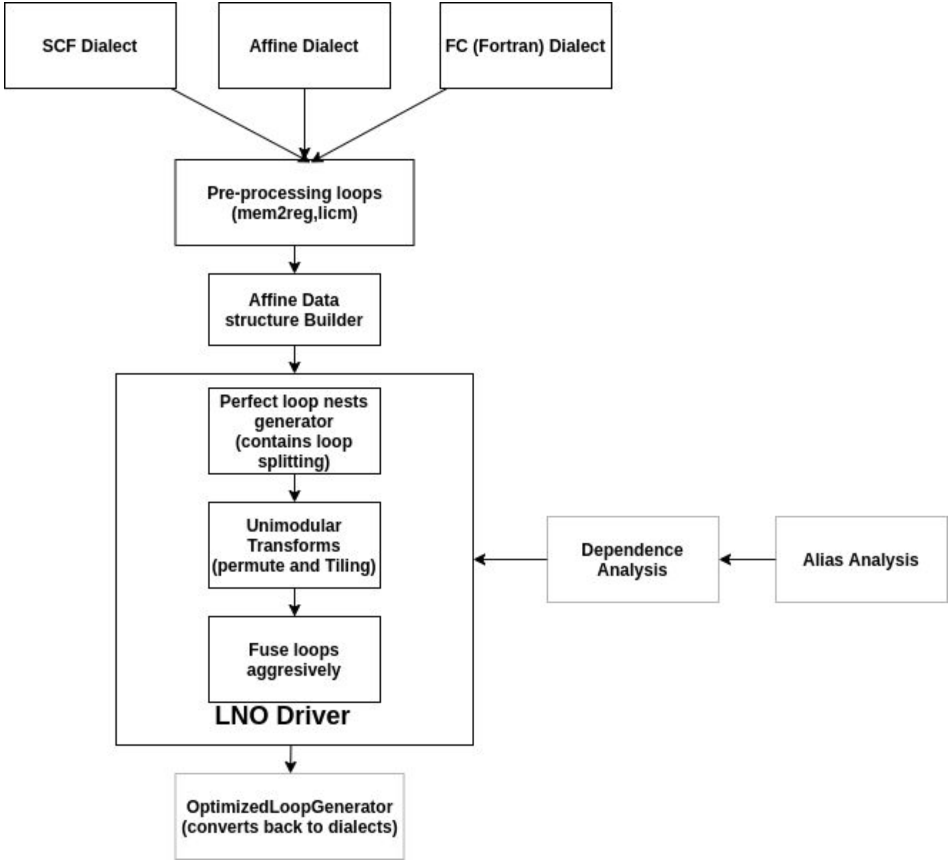
Not all Affine loops can be converted to Affine Ops

- In few cases, better to do loop transforms on higher level Dialects
    - Example: Fortran do loops with labels
- Restrictions on Affine Symbols and Dimensions
- All the loops in the loop nest may not be “affine.for”
- **Lower conversion rate to Affine Ops**
- AffineMap and AffineExpr can be freely used in custom Dialects

Heuristic Based Classic Loop Transformation Framework

- A proof-of-concept implementation of loop transformations along with a cost aware driver.
- Built as wrapper around Affine Dialect data structures
- Different Loop Transformations:
  - **Unimodular Transformations (Loop Permute and Loop Blocking)**
  - **Loop Fission, Loop Fusion**
- A basic profitability model based on cache utilization.
- **AliasAnalysis (basic-aa), Dependence Analysis**, etc ported from LLVM infrastructure
- **Mem2reg, licm**, etc as pre-processing steps
- Driver is currently written for **Data Locality** but it can be tuned for any custom workloads/ hardware.

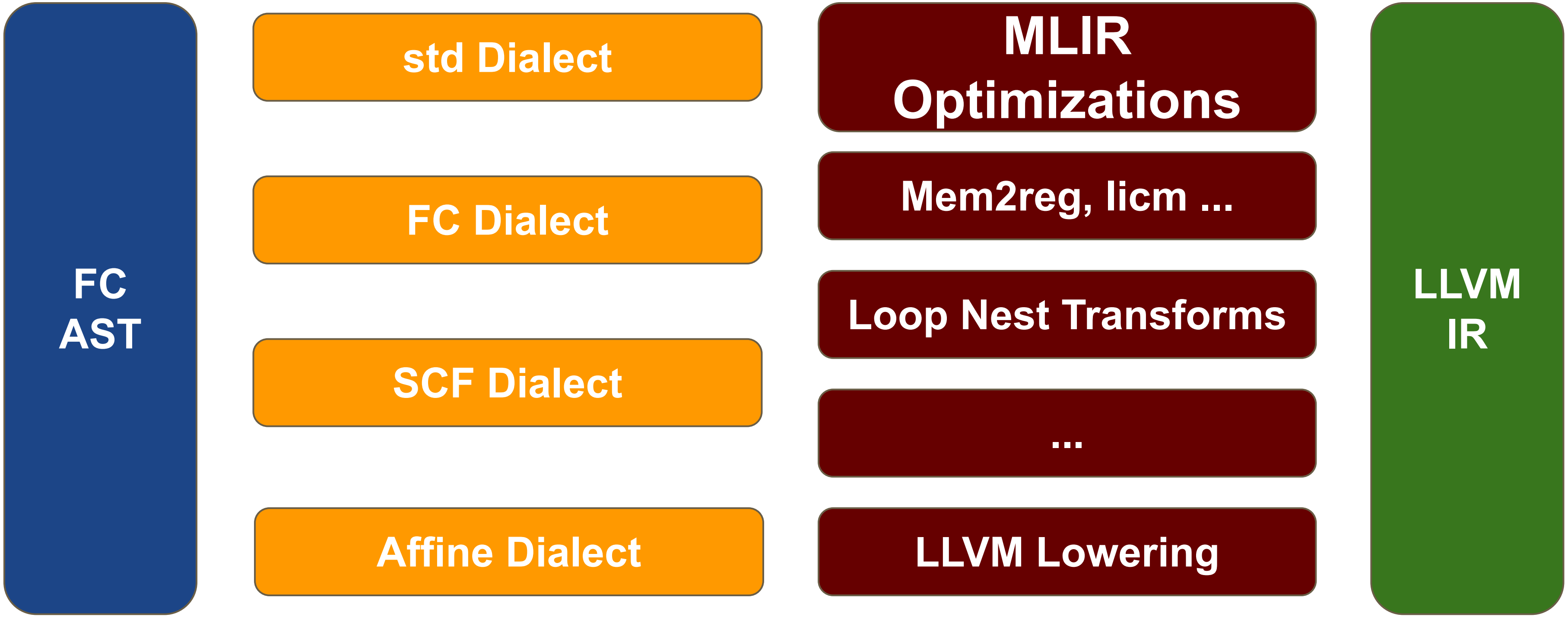
Pass Pipeline



Various inputs to the framework

- Focus on SPEC CPU 2017 benchmarks: Fortran / C++ / C
  - **Fortran Dialect: FC compiler**
    - Loop representations: *do, do while , forall, parallel do*
    - Array section operations: converted to affine.for
    - I/O operations
    - Various intrinsic functions
  - **CIL Dialect : C/ C++ representation in MLIR**
    - Low level IR (pointer type based)
    - Experimental path
- TODO: **Tensorflow XLA**
  - Affine loops generated from lhlo

FC and MLIR



MLIR Dialects

FC: Affine Dialect Conversion and Canonicalization of Loop Nests

Fortran 90:

```
do i = 1, 10
  do j = 1, 20
    b(i+1, j+2) = c(i, j)
  enddo
enddo
```

FC MLIR codegen →

Loop Dialect: **(sub-optimal IR)**

```
scf.for %arg0 = %4 to %7 step %6 {
    %c1_i32_3 = constant 1 : i32
    %8 = index_cast %c1_i32_3 : i32 to index
    ……
    scf.for %arg1 = %8 to %11 step %10 {
      %12 = load %1[%arg0, %arg1] {name = "c"} : memref<10x20xi32, #map
      %c1_i32_5 = constant 1 : i32
      %13 = index_cast %c1_i32_5 : i32 to index
      %14 = addi %arg0, %13 : index
      %c2_i32 = constant 2 : i32
      %15 = index_cast %c2_i32 : i32 to index
      %16 = addi %arg1, %15 : index
      store %12, %2[%14, %16] {name = "b"} : memref<11x22xi32, #map0>
    }
  }
```

← *Affine dialect converter*

Affine Dialect:

```
affine.for %arg0 = 1 to 11 {
    affine.for %arg1 = 1 to 21 {
      %4 = affine.load %1[%arg0, %arg1] : memref<10x20xi32, #map0>
      affine.store %4, %2[%arg0 + 1, %arg1 + 2] : memref<11x22xi32, #map0>
    }
  }
```

Analysis Passes

Alias and Dependence Analysis

- Alias Analysis
  - Generic Infrastructure for existing / custom Dialect memory operations
  - Invoked using AliasSetTracker (ported from LLVM)
  - Implemented by BasicAA and Dependence Analysis

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  - Ported from LLVM
  - Works on Affine data structures (mlir::AffineExpr)
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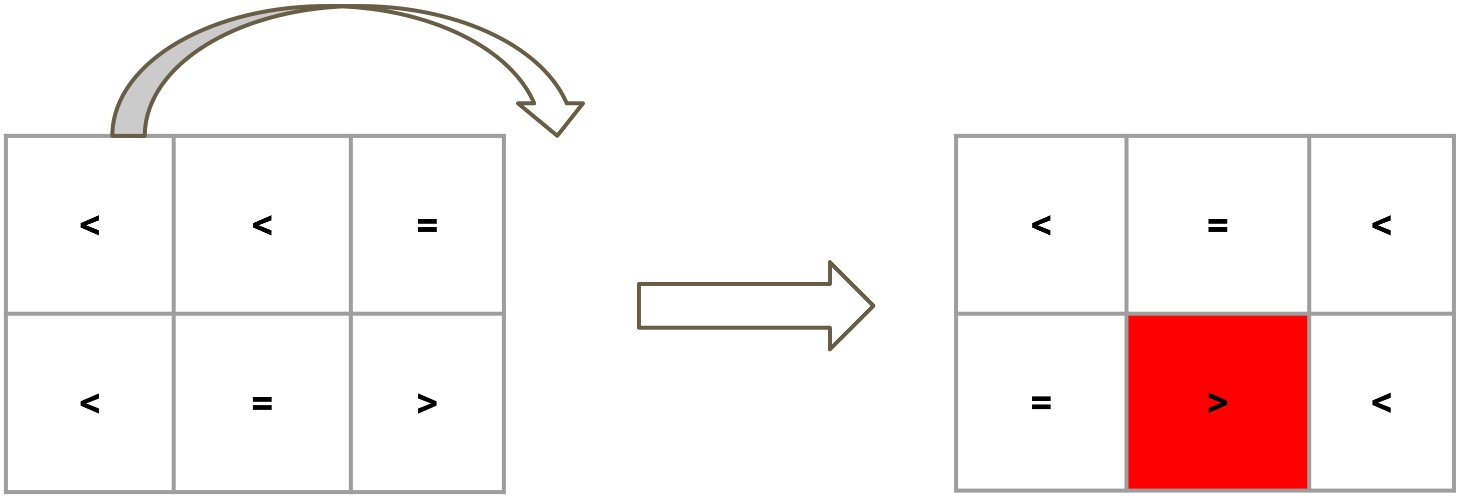
  - Ported from LLVM
  - Works on Affine data structures (mlir::AffineExpr)
  - Uses Alias Analysis
- Dependence Matrix
  - Wrapper on top of Dependence Analysis
  - Contains all the dependencies in the given loop nest.
  - Contains m x n dependence matrix, where ‘m’ is number of dependences and ‘n’ is number of loops in the nest

Dependency Matrix

```
for (int i = 0; i < n; ++i) {
  for (int j = 1; j < m; ++j) {
    for (int k = 1; k < l; ++k) {
      a[i+1][j+1][k] = a[i][j][k] + a[i][j+ 1][k+1];
    }
  }
}
```

<	<	=
<	=	>

Legality of transformation



Loop Cost Analysis

- Gives out a cost for each loop in its nest based on cache misses.
  - Permute, Split, Fuse, Blocking, Prefetching and other cache related opts can use this data.
- Loop Cost for each loop is calculated as follows:
  - A penalty is assigned to the loop based on the amount of cache misses it will cause to the references in the loop nest.
  - Group the references that belong to the same cache line and assign penalty,
    - If the reference is a “scalar” value with respect loop then penalty us **1**.
    - If the reference is a “strided” access w.r.t. the loop, then the penalty is **TripCount / CacheLineSize**
    - If the reference is a “non-strided” access w.r.t. the loop, then penalty is TripCount
  - Total Cost = Cost due to penalties x number of times the loop executes due to outer loops.
- Concerns:
  - Need to get CacheLineSize from Target to accurately calculate cost for a given processor.

Loop Cost

```
for (int i = 1; i < n; ++i) {
  for (int j = 1; j < n; ++j) {
    B[i][j+10]  += C[j][i] + D[i][j];
  }
}
```

B[i][j+10]	n / L
C[j][i]	n
D[i][j]	n/L
Total Cost	n (n + 2n/L)

Strided access for j loop

Non-contiguous access for j loop

Contiguous access for j loop

Loop Cost

```
for (int j = 1; j < n; ++j) {
  for (int i = 1; i < n; ++i) {
    B[i][j+10]  += C[j][i] + D[i][j];
  }
}
```

Non-contiguous access for loop i

Contiguous access for i

Non-contiguous access for i loop

B[i][j+10]	n
C[j][i]	n/L
D[i][j]	n
Total Cost	n (2n + n / L)

Pre-processing of Loop Nests

Pre-processing passes

- Helps in creating Perfect Loop Nests

- Promote Memory to Register (mem2reg):
    - Works similar to LLVM's mem2reg
    - Works on memrefs
    - No restriction on Alloca / Memory access operations (can be from affine / std, etc)
- Hoisting invariants (LICM):
    - Similar to LLVM's licm pass: Hoists invariants out of Loops
    - Uses Alias Analysis
- Sinking operations:
    - Tries to sink operations to innermost loop
    - Uses Alias Analysis
- Affine Normalization
    - Create one Affine map for the loop nest

Example

```fortran
subroutine foo(a, b, c)
  integer :: a(10, 10), b(10, 10), c(10, 10)
  integer :: i, j
  integer :: k

  do i = 1, 10
   k = i * i
    do j = 2, 10
      a(i, j) = b(i, j) + c(j, i) + k
    end do
  end do
end subroutine
```

```
module {
  func @foo(%arg0: !fc.ref<!fc.array<1:10 x 1:10 x i32>>, %arg1: ...) {
    %c2_i32 = constant 2 : i32
    %c10_i32 = constant 10 : i32
    %c1_i32 = constant 1 : i32
    %c11_i32 = constant 11 : i32
    %0 = fc.allocate j : !fc.ref<i32>
    %1 = fc.allocate k : !fc.ref<i32>
    fc.do %arg3 = %c1_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
      %2 = muli %arg3, %arg3 : index
      %3 = index_cast %2 : index to i32
      fc.store %3, %1 {name = "k"} : !fc.ref<i32>
      fc.do %arg4 = %c2_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
        %4 = fc.load %arg1[%arg3, %arg4]
        %5 = fc.load %arg2[%arg4, %arg3]
        %6 = addi %4, %5 : i32
        %7 = fc.load %1 {name = "k"} : i32
        %8 = addi %6, %7 : i32
        fc.store %8, %arg0[%arg3, %arg4]
      } enddo {construct_name = ""}
    } enddo {construct_name = ""}
    return
  }
}
```

Example

```
module {
  func @foo(%arg0: !fc.ref<!fc.array<1:10 x 1:10 x i32>>, %arg1: ...) {
    %c2_i32 = constant 2 : i32
    %c10_i32 = constant 10 : i32
    %c1_i32 = constant 1 : i32
    %c11_i32 = constant 11 : i32
    %0 = fc.allocate j : !fc.ref<i32>
    %1 = fc.allocate k : !fc.ref<i32>
    fc.do %arg3 = %c1_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
      %2 = muli %arg3, %arg3 : index
      %3 = index_cast %2 : index to i32
      fc.store %3, %1 {name = "k"} : !fc.ref<i32>
      fc.do %arg4 = %c2_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
        %4 = fc.load %arg1[%arg3, %arg4]
        %5 = fc.load %arg2[%arg4, %arg3]
        %6 = addi %4, %5 : i32
        %7 = fc.load %1 {name = "k"} : i32
        %8 = addi %6, %7 : i32
        fc.store %8, %arg0[%arg3, %arg4]
      } enddo {construct_name = ""}
    } enddo {construct_name = ""}
    return
  }
}
```

⇒

```
module {
  func @foo(%arg0: !fc.ref<!fc.array<1:10 x 1:10 x i32>>, %arg1: ...) {
    %c2_i32 = constant 2 : i32
    %c10_i32 = constant 10 : i32
    %c1_i32 = constant 1 : i32
    fc.do %arg3 = %c1_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
      fc.do %arg4 = %c2_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
        %5 = muli %arg3, %arg3 : index
        %6 = index_cast %5 : index to i32
        %7 = fc.load %arg1[%arg3, %arg4] ...
        %8 = fc.load %arg2[%arg4, %arg3] ...
        %9 = addi %7, %8 : i32
        %10 = addi %9, %6 : i32
        fc.store %10, %arg0[%arg3, %arg4]
      } enddo {construct_name = ""}
    } enddo {construct_name = ""}
    return
  }
}
```

Loop Transformation Driver

Loop Transformation Driver

- Generic framework
  - Works on affine / scf /user-defined dialect by writing converter
- Algorithm:
  - Aggressively split the loop nest across Statements and Sibling loops
  - Run pre-processing on the loop nests (if needed)
  - Run the unimodular transformations on the single perfect loop nest
  - Aggressively fuse the loops whenever feasible
- Loop Fusion and Unimodular Transformations are driven using profitability models

Creation of Perfect Loop Nests: Loop Splitting

- Recursively split the loop nests based on Dependence Analysis to generate Perfect Loop Nests
- Input to Unimodular Transforms

```
subroutine foo(a, b, c)
  integer :: a(10, 10), b(10, 10), c(10, 10)
  integer :: i, j

  do i = 1, 10
    c(i, i) = i * i
    do j = 2, 10
      a(i, j) = b(i, j) + c(i, j)
    end do
  end do
end subroutine
```

⇒

```
subroutine foo(a, b, c)
  integer :: a(10, 10), b(10, 10), c(10, 10)
  integer :: i, j

  do i = 1, 10
    c(i, i) = i * i
  end do

  do i = 1, 10
    do j = 2, 10
      a(i, j) = b(i, j) + c(i, j)
    end do
  end do
end subroutine
```

Example

```
module {
  func @foo(%arg0: !fc.ref<!fc.array<1:10 x 1:10 x i32>>, %arg1: ...) {
    %c2_i32 = constant 2 : i32
    %c10_i32 = constant 10 : i32
    %c1_i32 = constant 1 : i32
    %c11_i32 = constant 11 : i32
    %0 = fc.allocate j : !fc.ref<i32>
    fc.do %arg3 = %c1_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
      %1 = muli %arg3, %arg3 : index
      %2 = index_cast %1 : index to i32
      fc.store %2, %arg2[%arg3, %arg3] ...
      fc.do %arg4 = %c2_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
        %3 = fc.load %arg1[%arg3, %arg4] ...
        %4 = fc.load %arg2[%arg3, %arg4] ...
        %5 = addi %3, %4 : i32
        fc.store %5, %arg0[%arg3, %arg4] ...
      } enddo {construct_name = ""}
    } enddo {construct_name = ""}
    return
  }
}
```

```
module {
  func @foo(%arg0: !fc.ref<!fc.array<1:10 x 1:10 x i32>>, %arg1: ...) {
    %c2_i32 = constant 2 : i32
    %c10_i32 = constant 10 : i32
    %c1_i32 = constant 1 : i32
    fc.do %arg3 = %c1_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
      %1 = muli %arg3, %arg3 : index
      %2 = index_cast %1 : index to i32
      fc.store %2, %arg2[%arg3, %arg3] ...
    } enddo {construct_name = ""}
    fc.do %arg3 = %c1_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
      fc.do %arg4 = %c2_i32, %c10_i32, %c1_i32 {construct_name = ""}  {
        %1 = fc.load %arg1[%arg3, %arg4] ...
        %2 = fc.load %arg2[%arg3, %arg4] ...
        %3 = addi %1, %2 : i32
        fc.store %3, %arg0[%arg3, %arg4] ...
      } enddo {construct_name = ""}
      fc.store %c11_i32, %0 : !fc.ref<i32>
    } enddo {construct_name = ""}
    return
  }
}
```

Unimodular transformations

- Represented by a unimodular transformation matrix (determinant 1 or -1)
- Composition of loop permutation, skewing, reverse
- **T * i = i'**, T is the transformation matrix, i and i' are dependence matrices
- Transformation is legal if the transformed dependence matrix is lexicographically positive
- Eg: for permute of (2-d loop nest), **T =** $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$

Unimodular transformations

- Input: Perfect Loop Nests
- Analysis
  - Legality
  - Uses dependence analysis and then cost analysis on loop nest to output the optimal transformation matrix for the given loop nest
- Transformation
  - Generate loop bounds in transformed space
  - Perform **Fourier-motzkin elimination** to simplify the transformed bounds
  - Validate and update the loop bounds for all loops in the nest
  - Update all memory accesses
    1. Crate a map of old indvars -> new indvars
    2. Rewrite the accesses using the new indvars information

Example : Matrix Multiplication (for vectorization)

Aggressively apply splitting → unimodular transformations → fusion

```
for (int i = 1; i < n; ++i) {
  for (int j = 1; j < n; ++j) {
    A[i][j] = 0;
    for (int k = 1; k < n; ++k) {
      A[i][j] += B[i][k] * C[k][j];
    }
  }
}
```

Split

```
for (int i = 1; i < n; ++i)
  for (int j = 1; j < n; ++j)
    A[i][j] = 0;

for (int i = 1; i < n; ++i) {
  for (int j = 1; j < n; ++j) {
    for (int k = 1; k < n; ++k) {
      A[i][j] += B[i][k] * C[k][j];
    }
  }
}
```

Unimodular Transforms (Permute)

Loop Cost aids it.

```
for (int i = 1; i < n; ++i)
  for (int j = 1; j < n; ++j)
    A[i][j] = 0;

for (int i = 1; i < n; ++i) {
  for (int k = 1; k < n; ++k) {
    for (int j = 1; j < n; ++j) {
      A[i][j] += B[i][k] * C[k][j];
    }
  }
}
```

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Fuse

```
for (int i = 1; i < n; ++i) {
  for (int j = 1; j < n; ++j)
    A[i][j] = 0;
  for (int k = 1; k < n; ++k) {
    for (int j = 1; j < n; ++j) {
      A[i][j] += B[i][k] * C[k][j];
    }
  }
}
```

Loop Permutation

Innermost Loop (i is outermost)	A[i][j]	B[i][k]	C[k][j]	Total
j	n^3/b	n^2	n^3/b	$2n^3/b + n^2$
k	n^2	n^3/b	n^3	$n^3(1+1/b) + n^2$

```
for (int i = 1; i < n; ++i)
  for (int j = 1; j < n; ++j)
    A[i][j] = 0;

for (int i = 1; i < n; ++i) {
  for (int j = 1; j < n; ++j) {
    for (int k = 1; k < n; ++k) {
      A[i][j] += B[i][k] * C[k][j];
    }
  }
}
```

b is the cache line size for the target

Loop Permutation

j as innermost loop gives lesser cost!

Innermost Loop (i is outermost)	A[i][j]	B[i][k]	C[k][j]	Total
j	n^3/b	n^2	n^3/b	$\mathbf{2n^3/b + n^2}$
k	n^2	n^3/b	n^3	$n^3(1+1/b) + n^2$

```
for (int i = 1; i < n; ++i)
  for (int j = 1; j < n; ++j)
    A[i][j] = 0;

for (int i = 1; i < n; ++i) {
  for (int k = 1; k < n; ++k) {
    for (int j = 1; j < n; ++j) {
      A[i][j] += B[i][k] * C[k][j];
    }
  }
}
```

Loop Blocking

- Access data in blocks to exploit temporal and spatial locality
- Transform a loop at a depth into two loops:
  - One loop for iterating inside each block
  - One loop for iterating over the blocks
- Block size
  - fixed at compile time (each depth can have a different one)
  - depends on cache size and cache line size
  - determined by tuning
- Strip-mining and interchange

```
for (int i = 0; i < n; ++i)
  for (int j = 0; j < n; ++j)
    A[i] = A[i] + B[j];
```

Strip-mining

```
for (int i = 0; i < n; ++i)
  for (int j = 0; j < n; j+=B)
    for (int jj = j; jj < min(n, j+B-1); jj++)
      A[i] = A[i] + B[jj];
```

Interchange

```
for (int j = 0; j < n; j+=B)
  for (int i = 0; i < n; ++i)
    for (int jj = j; jj < min(n, j+B-1); jj++)
      A[i] = A[i] + B[jj];
```

Loop Blocking - matrix multiplication

```
for (int i = 0; i < n; ++i)
  for (int j = 0; j < n; ++j)
    for (int k = 0; k < n; ++k)
      A[i][j] += B[i][k] * C[k][j];
```

Cache misses for array B: n^3/b
Cache misses for array C: n^3

```
for (int kk = 0; kk < n; kk+=B)
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
      for (int k = kk; k < kk+B; ++k)
        A[i][j] += B[i][k] * C[k][j];
```

```
for (int jj = 0; jj < n; jj+=B)
  for (int kk = 0; kk < n; kk+=B)
    for (int i = 0; i < n; ++i)
      for (int j = jj; j < jj+B; ++j)
        for (int k = kk; k < kk+B; ++k)
          A[i][j] += B[i][k] * C[k][j];
```

```
for (int ii = 0; ii < n; ii+=B)
  for (int jj = 0; jj < n; jj+=B)
    for (int kk = 0; kk < n; kk+=B)
      for (int i = ii; i < ii+B; ++i)
        for (int j = jj; j < jj+B; ++j)
          for (int k = kk; k < kk+B; ++k)
            A[i][j] += B[i][k] * C[k][j];
```

Cache misses for array B: $B^2/b*n^3/B^3$
$= n^3/(Bb)$
Cache misses for array C: $B^2/b*n^3/B^3$
$= n^2/(Bb)$

Loop Blocking

- Transformation: given a Loop-Nest $L_0, \ldots L_k$
  - Strip-mine each L_i in consideration into $L_{i'}$ and $L_{i''}$
  - Move all $L_{i'}$ to outside
- Strip-mining is always legal
- Loop interchange not always legal
  - All loops in consideration must be safe to be moved outside
  - Each such loop must have only "=" or "<" in all the dependence vectors
- Profitability
  - Look for good reuse candidate in outer-loop iterations
    - should carry small-threshold dependencies of any type carried by the loop
    - loop index occurs with small stride in contiguous dimension, and in no other dimension
  - Need to account for misses because of the outer-strip loops (for the dependencies carried by the innermost loop)

Results

- We could transform the hot loop nest in **bwaves_r** SPEC CPU 2017 benchmark see decent gain.
- We see around 70% gain in matmul() kernel, etc

Next steps

- Add more Unimodular transformations
- Open source
- Integrate the Framework with TensorFlow XLA compiler
- Run more benchmarks

Thank You