Polyhedral Modeling for Heterogeneous Compute

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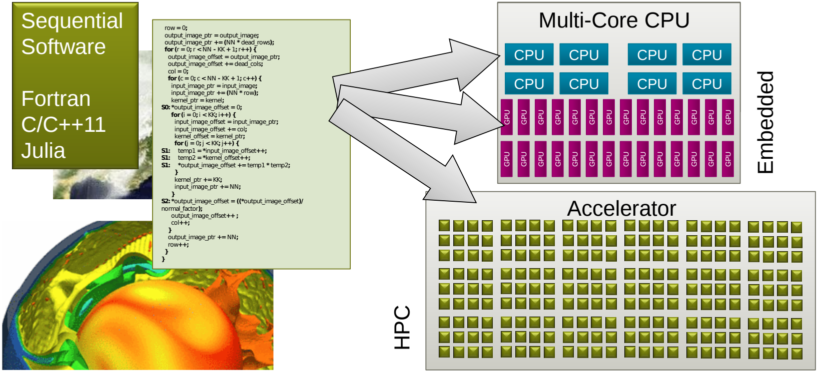
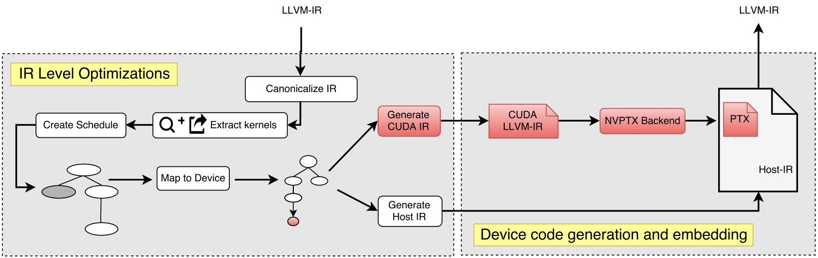
Objective


Sequential Software
Fortran
C/C++11
Julia
row = 0;
output_image_ptr = output_image;
output_image_ptr += (NN * dead_rows);
for (r = 0; r < NN - KK + 1; r++) {
output_image_offset = output_image_ptr;
output_image_offset += dead_cols;
col = 0;
for (c = 0; c < NN - KK + 1; c++) {
input_image_ptr = input_image;
input_image_ptr += (NN * row);
kernel_ptr = kernel;
S0: *output_image_offset = 0;
for (i = 0; i < KK; i++) {
input_image_offset = input_image_ptr;
input_image_offset += col;
kernel_offset = kernel_ptr;
for (j = 0; j < KK; j++) {
S1: temp1 = *input_image_offset++;
S1: temp2 = *kernel_offset++;
S1: *output_image_offset += temp1 * temp2;
}
kernel_ptr += KK;
input_image_ptr += NN;
}
S2: *output_image_offset = ((*output_image_offset)/
normal_factor);
output_image_offset++ ;
col++;
}
output_image_ptr += NN;
row++;
}
}
Multi-Core CPU
CPU CPU CPU CPU
CPU CPU CPU CPU
GPU
Embedded
Accelerator
HPC

Architecture



Mapping computations to thread-blocks and threads

```
for (i = 1; i <= 6; i++)
  for (j = 1, j <= 4; i++)
S:  B[i][j] =  A[i][j] + A[i+1][j  ] + A[i-1][j  ]
               + A[i  ][j+1] + A[i  ][j-1];
```

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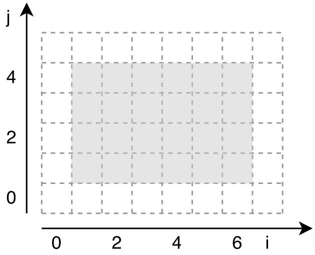

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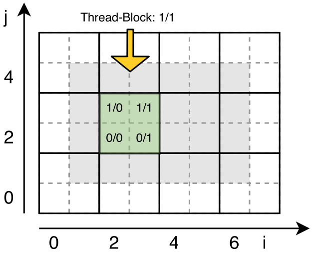

Mappings:

$$\{S[i,j] \rightarrow blocks[floor(i/2), floor(j,2)]\}$$
$$\{S[i,j] \rightarrow threads[i \bmod 2, j \bmod 2]\}$$

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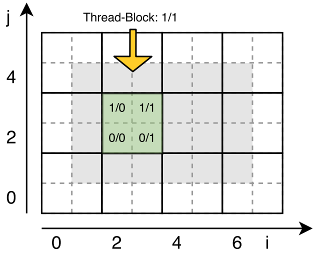


Mappings:

$\{S[i,j] \rightarrow blocks[floor(i/2), floor(j,2)]\}$
$\{S[i,j] \rightarrow threads[i \bmod 2, j \bmod 2]\}$

In case we create more thread-blocks than supported in hardware, thread-blocks are assigned round-robin!

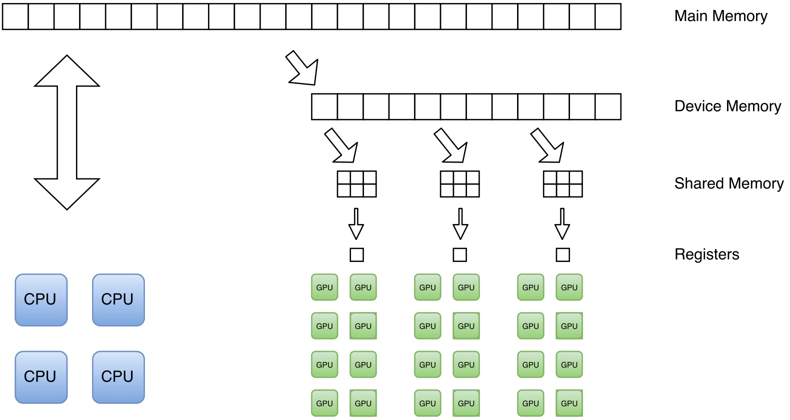
Generated accelerator code

```
void kernel(float A[][6], float B[][6]) {
  int b0 = blockIdx.y; int b1 = blockIdx.x;
  int t0 = threadIdx.y; int t1 = threadIdx.x;

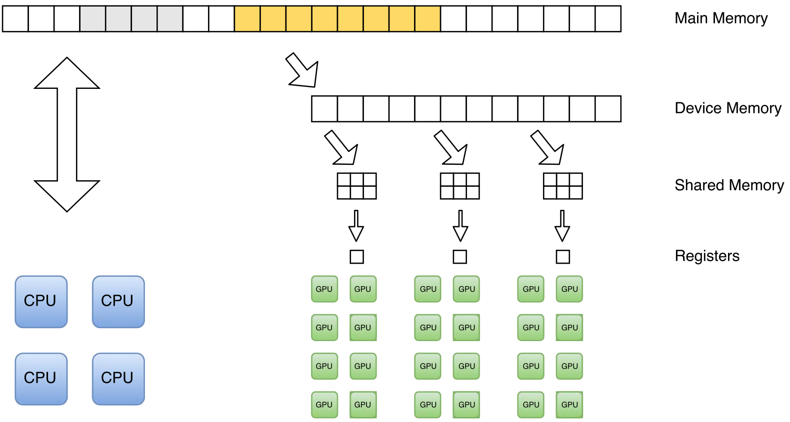
  int i = 2 * b0 + t0;
  int j = 2 * b1 + t1;
  S:  B[i][j] +=   A[i+1][j  ] + A[i-1][j  ]
                 + A[i  ][j+1] + A[i  ][j-1];
}
```

Commonly not a single computation per-kernel, but also loops/synchronizations.

Memory hierarchy of an accelerator system



Memory hierarchy of an accelerator system



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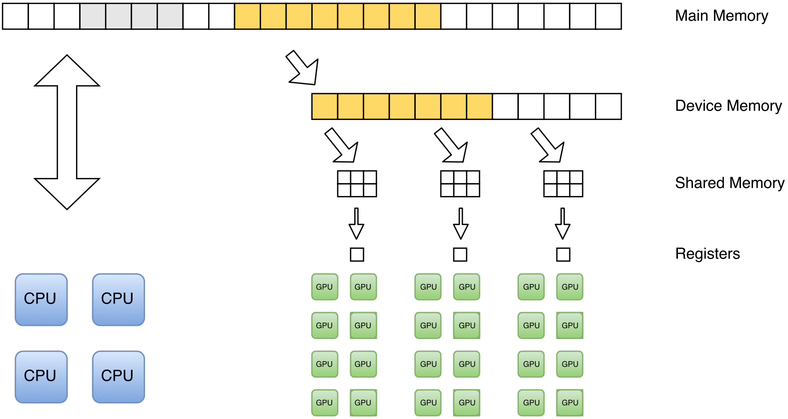

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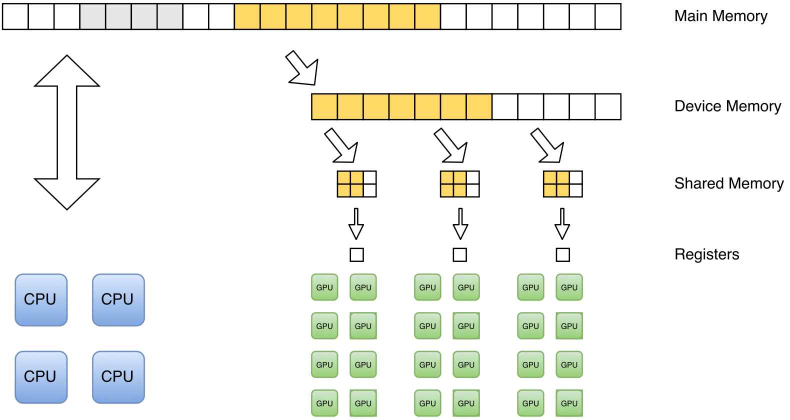

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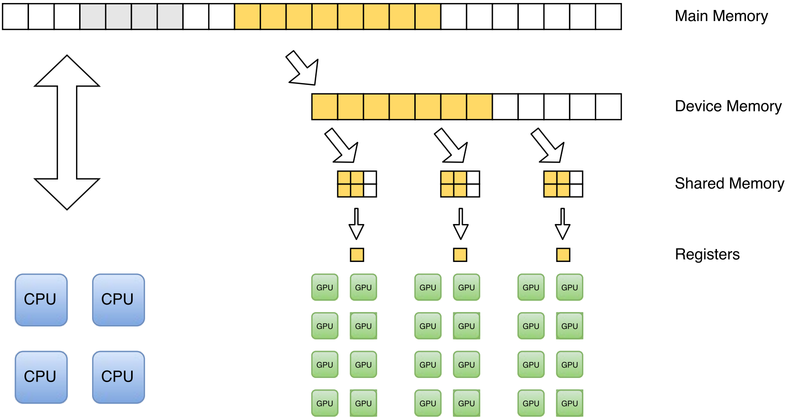
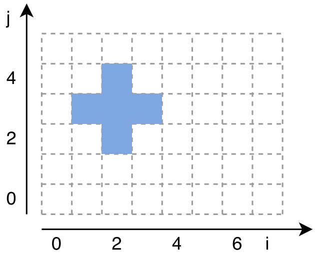


Identify array subregions accessed by threadblock

```
for (i = 1; i <= 6; i++)
  for (j = 1, j <= 4; i++)
S:  B[i][j] +=    A[i+1][j  ] + A[i-1][j  ]
            + A[i  ][j+1] + A[i  ][j-1];
```



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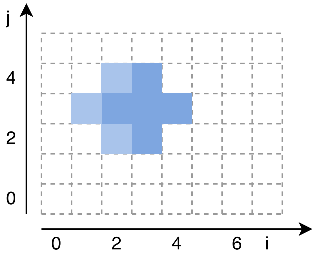

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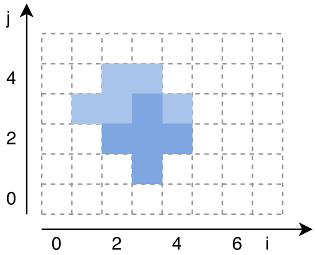

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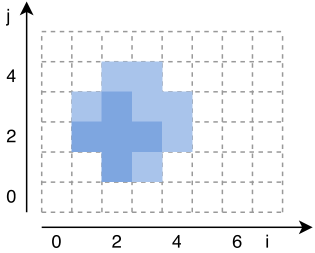

Maximal storage efficiency possible with counting (barvinok).
BUT, accesses become inefficient.

Identify array subregions accessed by threadblock

```
for (i = 1; i <= 6; i++)
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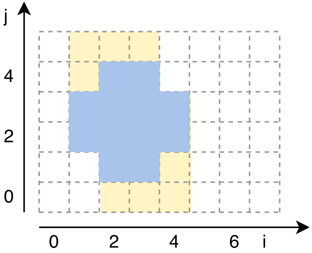
```



Copying a one-dimensional set of memory addresses (including untouched addresses in between).

Identify array subregions accessed by threadblock

```
for (i = 1; i <= 6; i++)
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S:  B[i][j] +=    A[i+1][j  ] + A[i-1][j  ]
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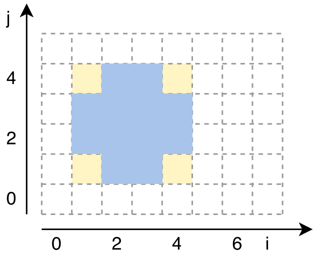
```



Copying a multi-dimensional set of array locations (including untouched addresses in between).

$\Rightarrow$ More efficient!

Map array subregions to shared memory

- For each array subregion identified, check if:
  - data-elements are used multiple times
    or
  - accesses to global memory are not coalesced
  - and the dataset size fits into shared memory

$\Rightarrow$ allocate shared memory for subregion

Generated code when using shared memory

Each thread-block executes:

- Copy global ⇒ shared (new)
- synchronize()
- Compute in shared memory (changed)
- synchronize()
- Copy shared ⇒ global (new)

Optimizing the copy code

Global ⇒ Shared

- Data element is read in thread-block
- … but has not been computed earlier in the same thread block
- Over approximate data to load with the rectangle to simplify code

Shared ⇒ Global

- Data element is written in thread-block
- … and is used later outside of the thread block but not overwritten in between.
- Do not over-approximate storage set.

Local memory / registers

- Algorithm mirrors shared memory mapping
- Use local memory in case data remains thread-local
- Unroll computation to ensure constant access expressions:

```
for (i = t0; i < 128; i+=32)
    A[floor(i / 32)] = i;

          ⇓

    A[0] = t0;
    A[1] = t0 + 32;
    A[2] = t0 + 64;
    A[3] = t0 + 96;
```

Lowering of arrays of parametric size in LLVM

```
void gemm(int n, int m, int p,
          float A[n][p], float B[p][m], float C[n][m]) {
L1:  for (int i = 0; i < n; i++)
L2:    for (int j = 0; j < m; j++)
L3:      for (int k = 0; k < p; ++k)
           C[i][j] += A[i][k] * B[k][j];
}
```

C99 arrays lowered to LLVM-IR

```
define void @gemm(i32 %n, i32 %m, i32 %p, float* %A, float* %B, float* %C) {
; for i:
;   for j:
;     for k:
        %A.idx = mul i32 %i, %p
        %A.idx2 = add i32 %A.idx, %k
        %A.idx3 = getelementptr float* %A, i32 %A.idx2
        %A.data = load float* %A.idx3
        %B.idx = mul i32 %k, %m
        %B.idx2 = add i32 %B.idx, %j
        %B.idx3 = getelementptr float* %B, i32 %B.idx2
        %B.data = load float* %B.idx3
        %C.idx = mul i32 %i, %m
        %C.idx2 = add i32 %C.idx, %j.0
        %C.idx3 = getelementptr float* %C, i32 %C.idx2
        %C.data = load float* %C.idx3
        %mul = fmul float %A.data, %B.data
        %add = fadd float %C.data, %mul
        store float %add, float* %C.idx3
}
```

Recovery of Index Expressions using SCEV

Recovered accesses are:

- Single dimensional
- *Polynomial*

```
void gemm(int n, int m, int p,
          float A[], float B[], float C[]) {
L1:  for (int i = 0; i < n; i++)
L2:    for (int j = 0; j < m; j++)
L3:      for (int k = 0; k < p; ++k)
           C[i * m + j] += A[i * p + k] * B[k * M + j];
}
```

The Problem

Given *a set of* **single dimensional memory accesses** *with index expressions that are multivariate polynomials and a set of iteration domains,* **derive a multi-dimensional view***:*

- A multi-dimensional array definition
- For each original array access:
  a new multi-dimensional access function

Grosser Tobias, Pop Sebastian, Pouchet Louis-Noel, Sadayappan P, Ramanujam J. **Optimistic Delinearization of Parametrically Sized Arrays**, International Conference on Supercomputing (ICS), 2015

Conditions

- **R1** - **Affine**
  New access functions are affine
- **R2** - **Equivalence**
  Addresses in original and multi-dimensional view are identical
- **R3** - **In-Bounds**
  Array subscripts are within bounds (except outer dimension)

If **R3** not statically provable $\rightarrow$ derive run-time conditions.

Example: Initialize subarray (I)

- Array size: $n_0 \times n_1 \times n_2$
- Subarray position: $o_0 \times o_1 \times o_2$
- Subarray size: $s_0 \times s_1 \times s_2$

```
void set_subarray(float A[],
                  size_t o0, size_t o1, size_t o2,
                  size_t s0, size_t s1, size_t s2,
                  size_t n0, size_t n1, size_t n2) {
  for (size_t i = 0; i < s0; i++)
    for (size_t j = 0; j < s1; j++)
      for (size_t k = 0; k < s2; k++)
S:        A[(n2 * (n1 * o0 + o1) + o2)
            + n1 * n2 * i + n2 * j + k] = 1;
      // A[o0 + i, o1 + j, o1 + k] = 1
}
```

Example: Initialize subarray (II)

1. **Start**
   $(n_2(n_1 o_0 + o_1) + o_2) + n_1 n_2 i + n_2 j + k$
2. **Expand expression**
   $n_2 n_1 o_0 + n_2 o_1 + o_2 + n_1 n_2 i + n_2 j + k$
3. **Extract Terms containing induction variables**
   $\{n_1 n_2 i, n_2 j, k\}$
4. **Drop non-parameters and sort terms by #elements**
   $\{n_1 n_2, n_2\}$
5. **Assumed size**
   `A[][n1][n2]`

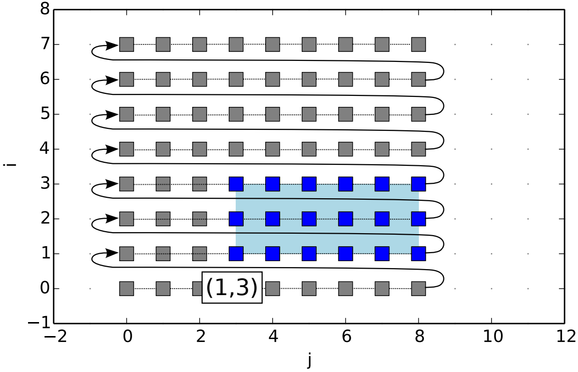
Example: Initialize subarray (III)

6. **Inner dimension**: divide by n_2
   Quotient: $n_1 o_0 + o_1 + n_1 i + n_2 j$
   Remainder: $o_2 + k$ $\rightarrow A[?][?][k + o_2]$
7. **Second inner dimension**: divide by n_1
   Quotient: $o_0 + i$ $\rightarrow A[i + o_0][?][?]$
   Remainder: $o_1 + j$ $\rightarrow A[?][j + o_1][?]$
8. **Full array access**: $A[i + o_0][j + o_1][k + o_2]$
9. **Validity conditions**:

$$\forall i, j, k : \quad 0 \leq i < s_0 \wedge 0 \leq j < s_1 \wedge 0 \leq k < s_2 :$$
$$0 \leq k + o_2 < n_2 \wedge 0 \leq j + o_1 < n_1 \wedge 0 \leq i + o_0$$
$$\Rightarrow o_1 \leq n_1 - s_1 \wedge o_2 \leq n_2 - s_2$$

Why validity conditions?

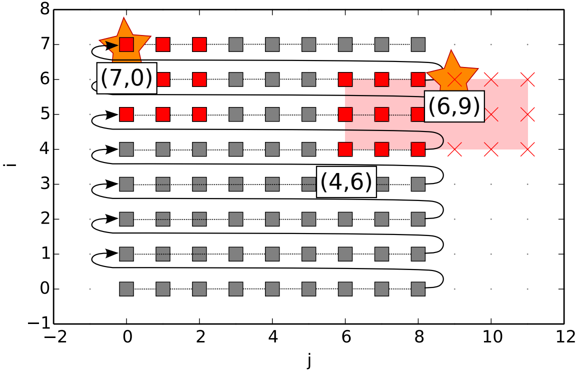
- Array size ($n_0 = 8, n_1 = 9$)
- Subarray offset ($o_0 = 1, o_1 = 3$), size ($s_0 = 3, s_1 = 6$).



- Run-time condition: $o_1 \leq n_1 - s_1 \Rightarrow 3 \leq 9 - 6 \rightarrow \top$

Why validity conditions?

- Array size ($n_0 = 8, n_1 = 9$)
- Subarray offset ($o_0 = 4, o_1 = 6$), size ($s_0 = 3, s_1 = 6$).



- Run-time condition: $o_1 \leq n_1 - s_1 \Rightarrow 6 \leq 9 - 6 \Rightarrow \bot$
- `A[6][9]` and `A[7][0]` alias ↯

Delinearization in LLVM's ScalarEvolution

```
// Delinearization of a single access
void delinearize(const SCEV *Expr,
    SmallVectorImpl<const SCEV *> &Subscripts,
    SmallVectorImpl<const SCEV *> &Sizes,
    const SCEV *ElementSize);

// Functions to derive a delinearization for a set of accesses:
void collectParametricTerms(const SCEV *Expr,
    SmallVectorImpl<const SCEV *> &Terms);
void findArrayDimensions(SmallVectorImpl<const SCEV *> &Terms,
    SmallVectorImpl<const SCEV *> &Sizes,
    const SCEV *ElementSize);
void computeAccessFunctions(
    const SCEV *Expr, SmallVectorImpl<const SCEV *> &Subscripts,
    SmallVectorImpl<const SCEV *> &Sizes);
```

! Validity conditions still need to be generated (available in Polly) !

Using shared memory: Apply a simple mapping function

```
void kernel(float A[][6], float B[][6]) {
  int b0 = blockIdx.y; int b1 = blockIdx.x;
  int t0 = threadIdx.y; int t1 = threadIdx.x;
  int i = 2 * b0 + t0; int j = 2 * b1 + t1;
  S:  B[i][j] =  A[i][j] + A[i+1][j  ] + A[i-1][j  ]
                         + A[i  ][j+1] + A[i  ][j-1];
}
```

Original access relation: $\{S[i,j] \to A[i,j]\}$

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                         + A[i  ][j+1] + A[i  ][j-1];
}
```

Original access relation: $\{S[i,j] \to A[i,j]\}$
Block mapping: $\{S[i,j] \to blocks[floor(i/2), floor(j,2)]\}$

Using shared memory: Apply a simple mapping function

```
void kernel(float A[][6], float B[][6]) {
  int b0 = blockIdx.y; int b1 = blockIdx.x;
  int t0 = threadIdx.y; int t1 = threadIdx.x;
  int i = 2 * b0 + t0; int j = 2 * b1 + t1;
  S:  B[i][j] =  A[i][j] + A[i+1][j  ] + A[i-1][j  ]
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}
```

Original access relation: $\{S[i,j] \rightarrow A[i,j]\}$

Block mapping: $\{S[i,j] \rightarrow blocks[floor(i/2), floor(j,2)]\}$

Per-block accesses: $\{blocks[b0, b1] \rightarrow A[i,j] \mid$

$$2 * b0 - 1 \leq i \leq 2 * b0 + 1 \wedge$$
$$2 * b1 - 1 \leq j \leq 2 * b1 + 1\}$$

Using shared memory: Apply a simple mapping function

```
void kernel(float A[][6], float B[][6]) {
  int b0 = blockIdx.y; int b1 = blockIdx.x;
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$$2 * b0 - 1 \leq i \leq 2 * b0 + 1 \wedge$$
$$2 * b1 - 1 \leq j \leq 2 * b1 + 1\}$$

Minimal element accessed in block: $(2b0 - 1, 2b1 - 1)$

Using shared memory: Apply a simple mapping function

```
void kernel(float A[][6], float B[][6]) {
  int b0 = blockIdx.y; int b1 = blockIdx.x;
  int t0 = threadIdx.y; int t1 = threadIdx.x;
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  S:  B[i][j] =  A[i][j] + A[i+1][j  ] + A[i-1][j  ]
                         + A[i  ][j+1] + A[i  ][j-1];
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Block mapping: $\{S[i,j] \rightarrow blocks[floor(i/2), floor(j,2)]\}$

Per-block accesses: $\{blocks[b0, b1] \rightarrow A[i,j] \mid 2*b0-1 \leq i \leq 2*b0+1 \wedge 2*b1-1 \leq j \leq 2*b1+1\}$

Minimal element accessed in block: $(2b0-1, 2b1-1)$

Extend of accessed region: $(3,3)$

Using shared memory: Apply a simple mapping function

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void kernel(float A[][6], float B[][6]) {
  int b0 = blockIdx.y; int b1 = blockIdx.x;
  int t0 = threadIdx.y; int t1 = threadIdx.x;
  int i = 2 * b0 + t0; int j = 2 * b1 + t1;
  S:  B[i][j] =  A[i][j] + A[i+1][j  ] + A[i-1][j  ]
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$2 * b0 - 1 \leq i \leq 2 * b0 + 1 \wedge$
$2 * b1 - 1 \leq j \leq 2 * b1 + 1\}$

Minimal element accessed in block: $(2b0 - 1, 2b1 - 1)$

Extend of accessed region: $(3, 3)$

Map to shared memory: $\{A[i,j] \rightarrow A_{\text{shared}}[i - 2b0 + 1, j - 2b1 + 1]\}$

Kernel code using shared memory

```
void kernel(float A[][6], float B[][6]) {
  int b0 = blockIdx.y; int b1 = blockIdx.x;
  int t0 = threadIdx.y; int t1 = threadIdx.x;
  __shared A_shared[3][3];

  A_shared[t0][t1] = A[2 * b0 + t0 - 1][2 * b1 + t1 - 1];
  if (t0 < 1)
    A_shared[t0+2][t1] = A[2 * b0 + t0 + 1][2 * b1 + t1 - 1];
  if (t1 < 1)
    A_shared[t0][t1+2] = A[2 * b0 + t0 - 1][2 * b1 + t1 + 1];
  if (t0 < 1 && t1 < 1)
    A_shared[t0+2][t1+2] = A[2 * b0 + t0 + 1][2 * b1 + t1 + 1];
  __sync_synchronize();
  S:  B[i][j] =   A_shared[t0+1][t1+1]
              + A_shared[t0+2][t1+1] + A_shared[t0+0][t1+1]
              + A_shared[t0+1][t1+2] + A_shared[i0+1][i1+0];
}
```

Heterogeneous Compute in Polly

- Precise memory modeling enables compiler-driven memory management.
- Polly recovers necessary information to reason about multi-dimensionality.
- Complex memory accesses transformations made easy.
- Sophisticated kernel generation with Polly