Prototyping a compiler for homomorphic encryption in MLIR

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Homomorphic Encryption?

- An encryption method that allows operations on ciphertexts (encryted texts)

- "Homomorphic": **Encrypt(x $*$ y) = Encrypt(x) $\circledast$ Encrypt(y)**

  +, *, ReLU, … on encrypted data: ***"Private AI"***

- Among 5 Impactful Emerging Technologies in 2022 from Gartner*

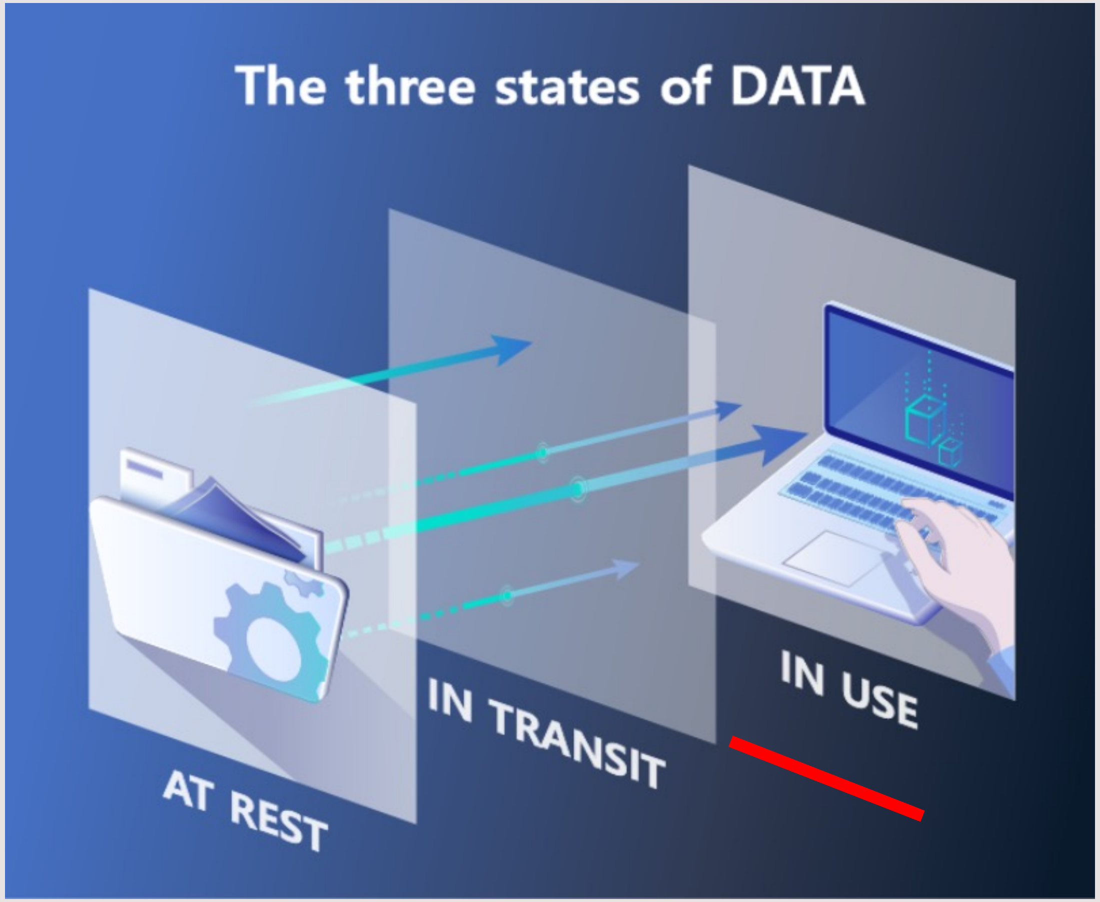
* https://www.gartner.com/en/articles/5-impactful-technologies-from-the-gartner-emerging-technologies-and-trends-impact-radar-for-2022

Why is HE Important?

It completes protection of the data

The three states of data

1. At Rest: Secure storage
2. In Transit: HTTPS
3. In Use: ***<u>HE</u>***



- HE Libraries: CryptoLab's HEaaN, Microsoft's SEAL, Duality Technologies' Palisade, …
- Layers using HE Libraries: IBM's HELayer, AWS's HIT, …

Challenge: Space & Speed

Homomorphic Encryption 101 – CKKS scheme

m: Message
double[N]

↔ Encode, Decode
(performs FP to int trunc.)

p: Plaintext
IntPoly

↔ Encrypt, Decrypt

c: Ciphertext
pair<IntPoly, IntPoly>

- IntPoly: A polynomial $a_0 + a_1 x + \cdots + a_{N-1} x^{N-1}$ with large integer coefficients
  - $0 \leq a_i < Q$ where Q is a large integer
  - To avoid using BigInt, $Q = q_0 \times q_1 \times \cdots \times q_{L-1}$ where q_j prime (uint64_t)
  - Coefficient a_i : uint64_t[L] which is $(a_i \ \% \ q_j)$ - called Residual Number System
  - All polynomial operations are modulo $(x^N + 1)$ (“polynomial ring”)

Challenge: Space & Speed

Homomorphic Encryption 101 – CKKS scheme

$\times L$ **or more** | $\times 2$

m: Message	⟷	p: Plaintext	⟷	c: Ciphertext
double[N]	Encode, Decode (performs FP to int trunc.)	IntPoly	Encrypt, Decrypt	pair<IntPoly, IntPoly>

- IntPoly: A polynomial $a_0 + a_1 x + \cdots + a_{N-1} x^{N-1}$ with large integer coefficients
- In C++, IntPoly is **uint64_t[N][L]**, or equiv. **uint64_t[L][N]**, where

  1. **N: the degree of the polynomial (~2^17)**
  2. **L: # of prime numbers used to represent coefficients (~30)**

Challenge: Space & Speed

Homomorphic Encryption 101 – CKKS scheme (En/Decryption)

p: Plaintext IntPoly	⟷ Encrypt, Decrypt	c: Ciphertext pair<IntPoly, IntPoly>

- Given a secret key s: IntPoly
- Enc(p, s): Ciphertext = **(a, -a*s + p + e)** where

  a: fresh random poly e: fresh error poly (random poly with small coeffs)

- Dec(c, s): Plaintext = c.second + c.first*s ~= **p + e**

Challenge: Space & Speed

Homomorphic Encryption 101 – CKKS scheme (En/Decryption)

p: Plaintext
IntPoly

Encrypt,
Decrypt

c: Ciphertext

1. Adding two polynomials is fast

```
for (i = 0 to N)
  for (j = 0 to L)
    res[i][j] = (as[i][j] + p[i][j]) % prime[j]
```

- Given a secret key s: IntPoly
- Enc(p, s): Ciphertext = **(a, -a*s + p + e)** where

  a: fresh random poly   e: fresh error poly (random poly with small coeffs)
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Challenge: Space & Speed

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- Dec(c, s): Plaintext = c.second + c.first*s

2. Multiplying two polynomials is slow! ☹

Naive product requires O(N^2)

Sol: Number-theoretic transformation!
Analogous to FFT
Time complexity: O(N log N)

Challenge: Space & Speed

Homomorphic Encryption 101 – CKKS scheme (Other Ops)

1. **Add(ctxt1, ctxt2): ctxt1 + ctxt2**
2. **Mult(ctxt1, ctxt2): ctxt1 * ctxt2**
   - Needs to remove s^2 term: 'evaluation key' must be <u>pre-calculated</u> & used
   - A scale factor multiplied by encode() is multiplicatively increased: 'rescale' op
3. **Rotate(ctxt, i): [ctxt[i], ctxt[i+1], …, ctxt[0], .., ctxt[i-1]]**
   - Needs 'rotation keys' that are <u>pre-calculated</u>
4. **Bootstrap(ctxt): very slow**
   - ctxt cannot be used after ~L multiplications; bootstrap revives it

Challenge: Space & Speed

In a Nut Shell

1. **Space**
   - Message size is multiplied by # of primes
   - If a message is not packed (m.size() < N), the factor is worse
   - mult, rotate, … requires pre-calculated keys that are large
2. **Speed**
   - Performs a lot of 64-bit (and sometimes 128-bit) int operations
   - Time complexity may be larger than O(N)
   - Even if O(N), it has large constant factors (L, multiple polynomials, …)

Challenge++: HE Runs on Diverse Environments!

1. **It must be fast on diverse environments**
    - On-premise is beneficial because *op(ctxt, **<u>ptxt2</u>**)* is faster than *op(ctxt, **<u>ctxt2</u>**)*
    - Encryption & decryption *<u>must</u>* be done on the device

2. **Utilizing both GPUs and CPUs brings benefits**
    - GPUs are fast but less cost-effective than deep learning (no FP ops)
    - High-end server CPUs have many cores & good at int benchmarks
    - Ciphertexts are large: sending them to GPU is intensive
    - Pre-calculated keys may not fit in GPU memory!

... but HE Library Developers Are Busy!

How to efficiently pack a matrix in ciphertexts?

How to reduce the error of approximated log x?

How to use less bootstrap operations?

.....

Can we use compiler optimization techniques to help them?

HEaaN.MLIR

Poly Lang → CPU (OpenMP) + GPU

- Src lang - **Poly**: a new lang for high-level operations on polynomial rings
- Tgt lang - {x86-64, AArch, …} (+OpenMP) x {CUDA, ROCm, …}
- In a developing stage; being prototyped using MLIR
- Currently, **HEaaN.MLIR** can:
  - ✓ **Compile encode+encryption (symm. & pub. key) and decryption written in Poly**
  - ✓ **Provide OpenMP offloading (works well) and CUDA (primitive)**

The Poly Lang

The HE parameter: uses predefined N, L, moduli, …

Polynomial with 30 moduli, NTT conversion applied

Polynomial with 30 moduli, no NTT

```
module attributes { poly.he_param = "FVa" } {
func @encrypt(%m: !poly.poly_ntt<30>, %s: !poly.poly_ntt<30>) -> (!poly.poly_ntt<30>, !poly.poly_ntt<30>) {
    // (a, −a*s + m + e)
    %e0 = poly.sample_gaussian (): !poly.poly<30>
    %e = poly.forward_ntt %e0: !poly.poly<30> to !poly.poly_ntt<30>
    %a0 = poly.sample_uniform (): !poly.poly<30>
    %a = poly.forward_ntt %a0: !poly.poly<30> to !poly.poly_ntt<30>

    %as = poly.mult_ntt %a, %s : !poly.poly_ntt<30>
    %asm = poly.sub_ntt %m, %as : !poly.poly_ntt<30>
    %b = poly.add_ntt %asm, %e : !poly.poly_ntt<30>
    return %a, %b: !poly.poly_ntt<30>, !poly.poly_ntt<30>
}
}
```

Q: %m is poly after encoding; how to do encoding?

The **Poly** Lang

The message (uses `complex` type in MLIR)

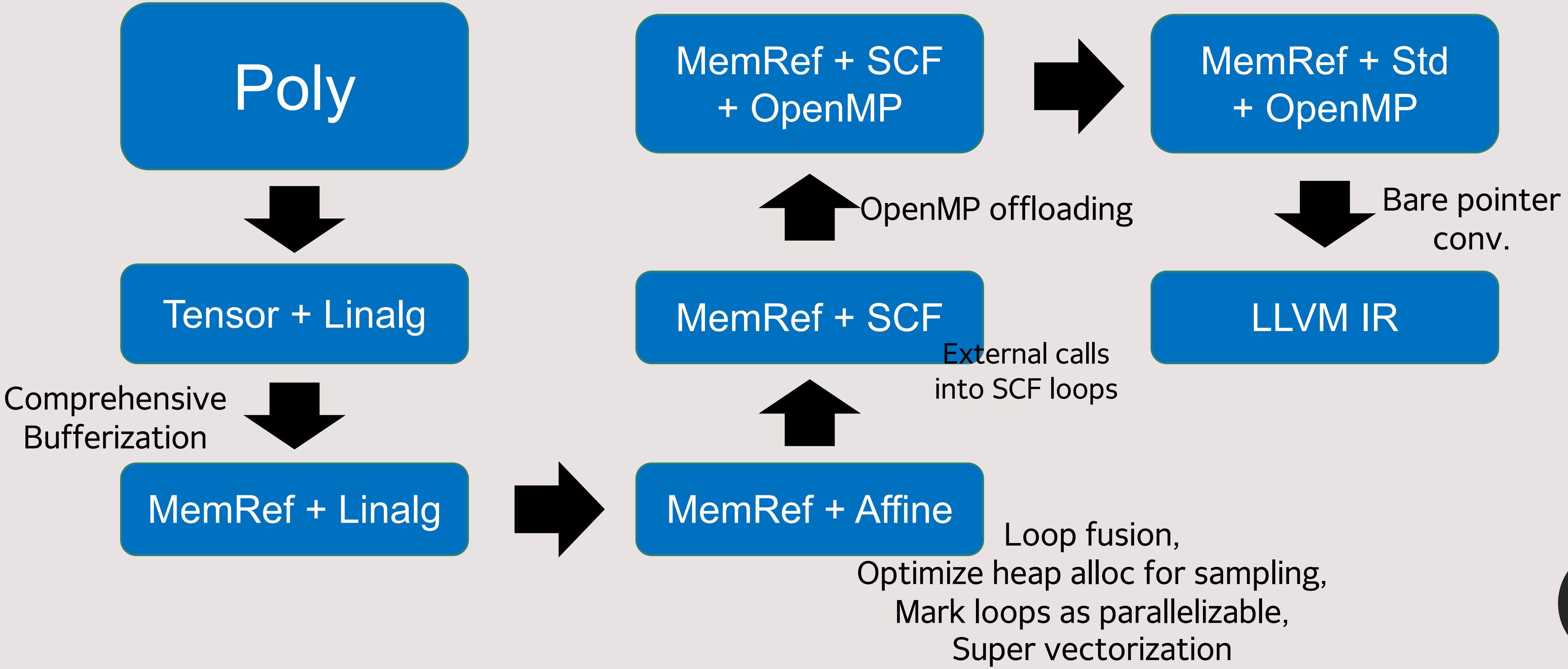
```
module attributes { poly.he_param = "FVa" } {
func @encode_without_ntt(%msg: tensor<65536xcomplex<f64>>) -> !poly.poly<30> {
    %y = poly.from_msg %msg: tensor<65536xcomplex<f64>> to !poly.poly<30>
    return %y: !poly.poly<30>
}
}
```

Now, you can write encode + encrypt!

Poly-to-tensor

- **!poly.poly<L>** and **!poly.poly_ntt<L>** are lowered into **tensor<LxNxi64>**
- tensor<..> and other types are kept intact
- **Poly** ops are lowered into **Linalg** + **Tensor** ops
- Constant tensors that are necessary for (efficient) calculation are inserted
- Operations that cannot be expressed in Linalg are temporarily represented as external fn calls!
    - Ex: NTT conversion loop: cannot be represented in Linalg.generic's reduction loop
    - Simply insert `call @__external_forward_NTT(..)` & lower it into SCF at a later pass

Pipeline of HEaaN.MLIR for CPU Code Gen



Loop Fusion: Collects Low-Hanging Fruits

- Benefit 1: Reduces the size of working sets by L in best cases.
- Benefit 2: Facilitates memory optimization → removes dead heap allocs
- Benefit 3: Removes synchronization points of OpenMP offloaded loops

```
for (i = 0 to L)
  for (j = 0 to N)
    B[i][j] = op(A[i][j]);

// sync. barrier

for (i = 0 to L)
  for (j = 0 to N)
    C[i][j] = op’(B[i][j]);
```

```
// B is now dead alloc.
for (i = 0 to L)
  for (j = 0 to N)

    C[i][j] =
       op’(op(A[i][j]));
```

Implementing Algorithms in MLIR

- Forward/backward NTT, FFT
- Barrett reduction
- Random sampling, ZO sampling, Gaussian sampling
- Many loops that are specifically necessary for encoding/decoding

Special thanks to Woosung for doing a lot of things from these!!

For Better Debugging Experience

- It is tricky to debug the generated code in terms of correctness & performance.
- To facilitate debugging, we:

1. Defined a **debug** dialect and used it: assertion, printer, timer

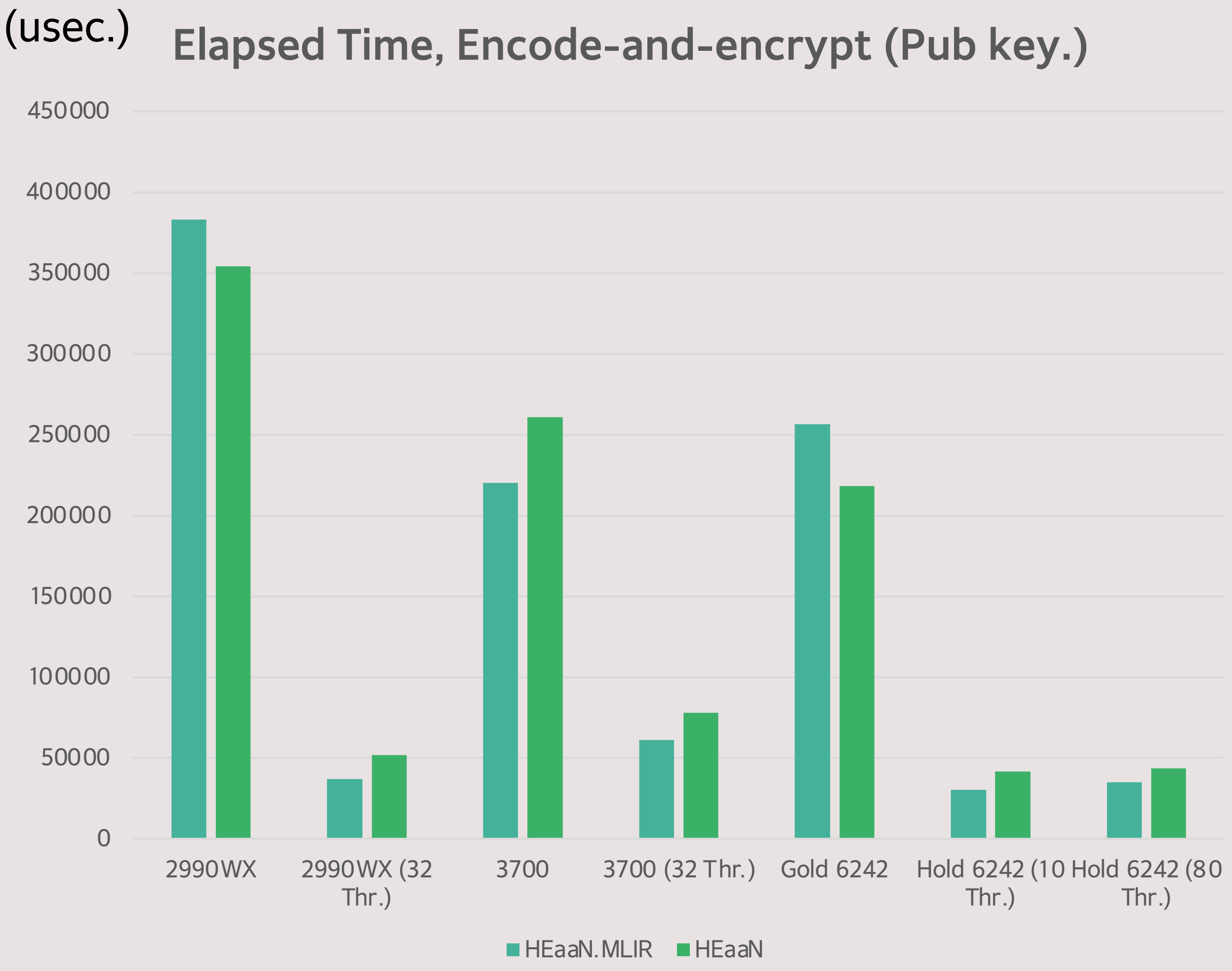
```
"debug.assert_eq"(%x, %y) {msg = "x and y must be equal"}: (i64, i64) -> ()
```

2. Added a '**sanitizer**' mode: insert bounds-checking assertions whenever creating memory accessing ops (memref.load/store)

Experimental Results

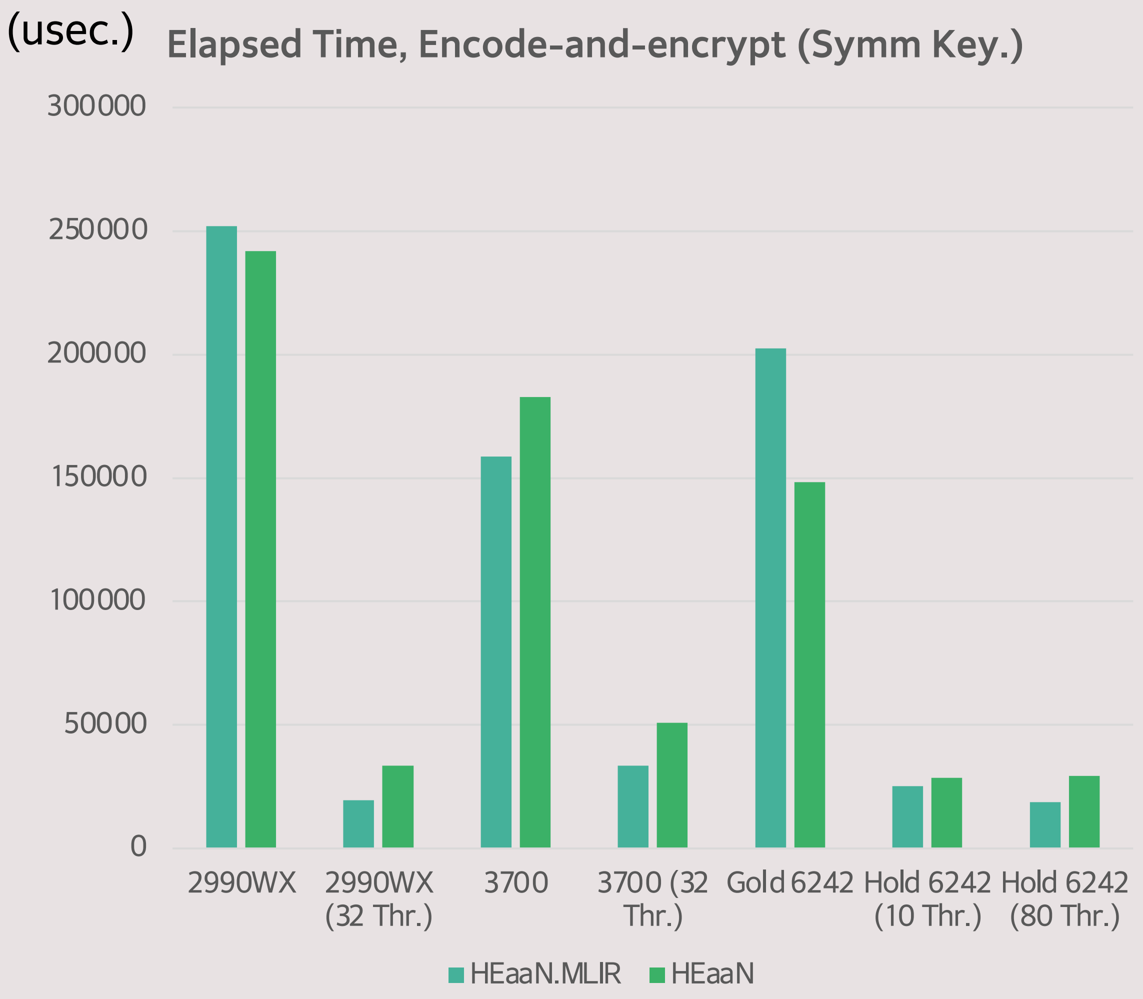
- Competitor: **HEaaN** (CryptoLab's proprietary HE library, use HEXL)
- 3 processors: **AMD Ryzen 2990WX, AMD Ryzen 3700, 2 Intel(R) Xeon(R) Gold 6242s**
- # Threads: 1 vs. full cores (for Gold: 1 vs. 10 vs. 80)
- Ran 50 times & calculated averages
- Disabled ASLR, set CPU to performance mode, ...

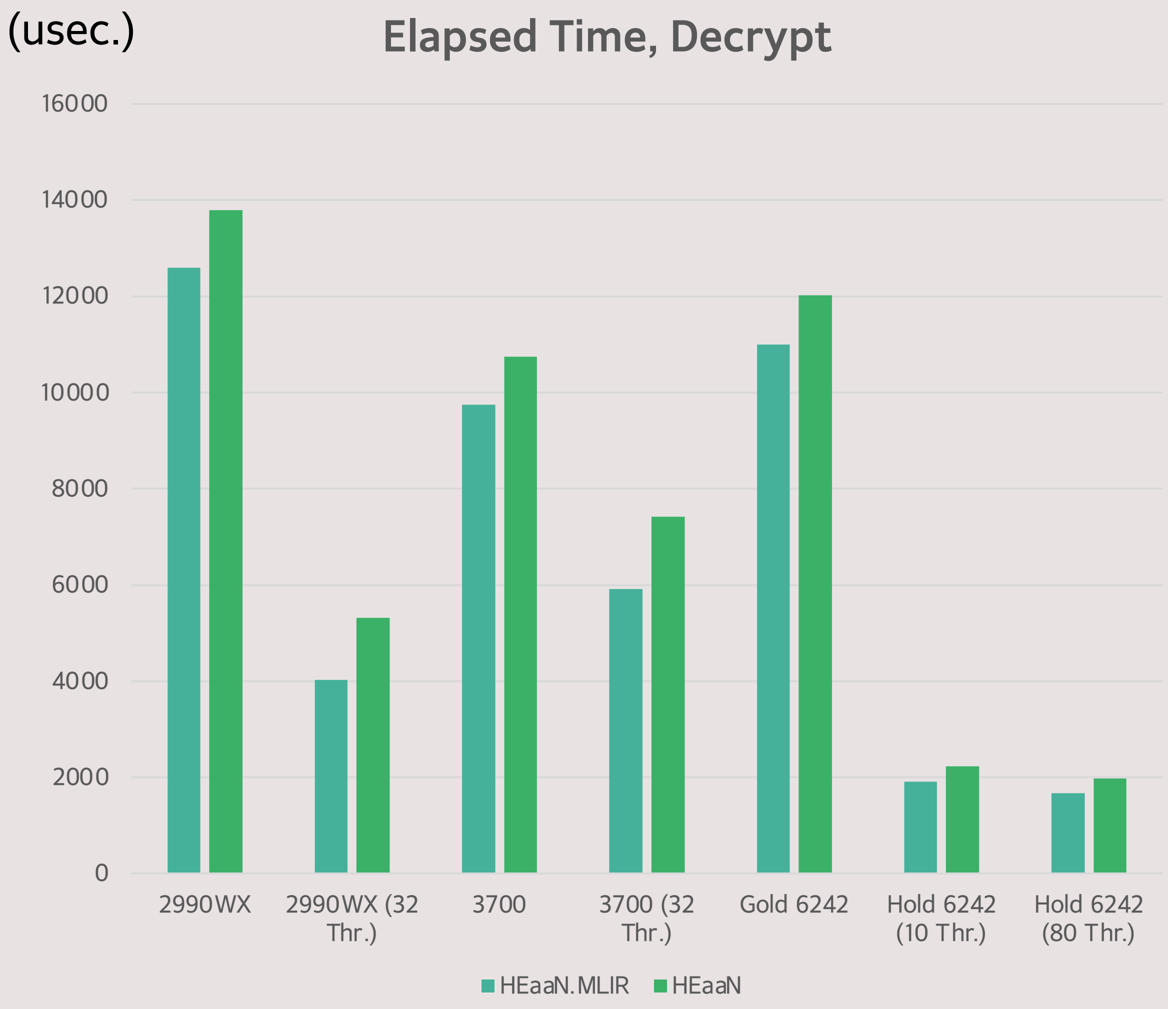
Experimental Results



- Single core results
  - Performance benefit was not clear
  - Gold 6242 has AVX512DQ: Intel HEXL gets benefit
- Multi core results
  - Parallelization was successful
  - Consistently got 40% speedups!

Experimental Results





Future Works

- **Faster NTT conversion**: directly invoke Intel HEXL if beneficial
- **Fully enable GPU offloading**: utilize GPUs in smartphones for en/decryption
- **Support more HE ops**: primary target is **rotation**!
  - For some benchmarks, about ½ of running time of matmul in HE is from rotations.
- **Correctness of compilation**: can we formally verify it?
  - SMT-based validation of transformations on structured loops seems to work well.
  - MLIR-TV*: another on-going (personal) project

* https://github.com/aqjune/mlir-tv

Thank you!